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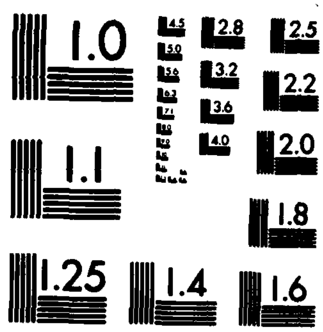

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NONPARAMETRIC DETECTION OF NARROWBAND SIGNALS*

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ABSTRACT

For nonparametric detection of narrowband signals in narrowband noise the zero medians assumption of the low-pass known-signal detection problem becomes the zero marginal medians assumption on the in-phase and quadrature noise components. Detectors based on conditional tests are defined for this problem which may be considered as being the most logical counterparts of the low-pass sign correlator detector. In addition, the symmetry assumption on the noise probability densities in the univariate case becomes a diagonal symmetry assumption, for which conditional multi-level nonparametric detectors are defined.

INTRODUCTION

For detection of a completely known low-pass signal in additive noise the sign correlator detector is a simple and useful detector for nonparametric detection, requiring only the assumption that the independent observations have zero medians under the noise-only hypothesis. Multi-level extensions of the sign correlator detector, based on conditional tests, can also be defined [1] to give more efficient nonparametric detection under a symmetry assumption on the noise probability density functions (pdf's). By a nonparametric detector we will mean one which for any finite sample size has a constant false-alarm probability under the noise-only hypothesis, for some nonparametric class of noise pdf's.

In this paper we will examine some detectors which may be viewed as counterparts of the sign correlator detector, for detection of coherent and incoherent narrowband signals in additive narrowband noise. Our objective is to define a narrowband detector which might be considered to be the counterpart of the univariate sign detector. We shall also see that an extension to multi-level detectors is also possible for such a detector, in exactly the same way that the multi-level sign correlator detector may be defined for known low-pass signal detection in symmetrically distributed noise. These narrowband detectors turn out to require conditional test implementations, as we will see.

The usual representation of an incoherent narrowband signal observed in additive narrowband noise $W(t)$ is

$$X(t) = \theta\, v(t) \cos[\omega_0 t + \phi(t) + \psi] + W(t) \qquad (1)$$

where $v(t)$ and $\phi(t)$ are known low-pass modulations, ψ is a random phase angle uniformly distributed on $[0, 2\pi]$, and the noise process $W(t)$ is stationary, zero-mean, bandpass white noise. The observation vectors processed by a detector are assumed to be the vector $\underline{X}_I$ and $\underline{X}_Q$ of uniformly sampled in-phase and quadrature components X_{Ii} and X_{Qi} respectively, $i = 1, 2, \ldots, n$. We have

$$X_{Ii} = \theta\, (s_{Ii} \cos\psi + s_{Qi} \sin\psi) + W_{Ii} \qquad (2a)$$

and

$$X_{Qi} = \theta\, (-s_{Ii} \sin\psi + s_{Qi} \cos\psi) + W_{Qi}. \qquad (2b)$$

where s_{Ii} and s_{Qi} are samples of $v(t)\cos\phi(t)$ and $-v(t)\sin\phi(t)$, respectively. An alternate statement of (2a) and (2b) is

$$\tilde{X}_i = \theta \tilde{s}_i\, e^{-j\psi} + \tilde{W}_i, \qquad (3)$$

in which in general we define $\tilde{A}_i = A_{Ii} + jA_{Qi}$. Note that for the completely known narrowband signal detection problem we have $\psi = 0$ in (1) and (3). In both cases under the null (noise-only) hypothesis H we have $\theta = 0$ and under the alternative K we have $\theta > 0$.

When $\theta = 0$ the observations are $(X_{Ii}, X_{Qi}) = (W_{Ii}, W_{Qi})$. Let f_{IQ} be the common bivariate probability density function (pdf) of the noise terms (W_{Ii}, W_{Qi}), $i = 1, 2, \ldots, n$. The general assumption is that these bivariate samples are independent random samples in addition to being *identically distributed*. The assumption on $W(t)$ also makes W_{Ii} and W_{Qi} uncorrelated random variables for each i. In order to obtain nonparametric detectors with a constant false-alarm probability under H some further assumptions need to be made about f_{IQ}.

NONPARAMETRIC CLASSES OF BIVARIATE NOISE DENSITIES

In the case of nonparametric detection of a known low-pass signal in additive noise a common assumption made about the noise distribution function is that its median value is zero. This makes the sign correlator detector a constant

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false-alarm probability detector. A further assumption which may be made to get more efficient detectors such as multi-level and signed-rank detectors is that of *symmetry* of the noise pdf. We will now consider some corresponding conditions which may be imposed on the bivariate noise pdf f_{IQ}.

Let F_{IQ} be the distribution function corresponding to f_{IQ}. A simple extension of the zero medians assumption in the univariate case is the zero marginal medians assumption

$$F_{IQ}(0, \infty) = F_{IQ}(\infty, 0) = 1/2 \quad ; \tag{4}$$

we shall be interested in defining a "sign" detector in the narrowband situation which is nonparametric in its performance, under H, for this assumption on F_{IQ}. An additional assumption which may be reasonable is that

$$F_{IQ}(0, 0) = 1/4 . \tag{5}$$

This condition together with (4) may be described as an equal-probability quadrants assumption.

A direct extension of the univariate symmetry assumption is that of diagonal symmetry in the bivariate case. For f_{IQ} this becomes

$$f_{IQ}(u, v) = f_{IQ}(-u,-v), \text{ all } u, v. \tag{6}$$

Diagonal symmetry implies that the marginal medians are zero. A stronger symmetry assumption is that of rectangular symmetry (component-wise symmetry), under which

$$f_{IQ}(u, v) = f_{IQ}(u, -v) = f_{IQ}(-u, v), \text{ all } u, v. \tag{7}$$

One can add an interchangeability (permutation invariance) assumption which requires

$$f_{IQ}(u, v) = f_{IQ}(v, u), \text{ all } u, v . \tag{8}$$

Note that diagonal symmetry together with interchangeability may be a reasonable assumption about f_{IQ}, but does not guarantee that W_{Ii} and W_{Qi} are uncorrelated. The rectangular symmetry assumption does imply this property for (W_{Ii}, W_{Qi}). We also find that with rectangular symmetry the equal-probability quadrants assumption [(4) and (5)] is satisfied.

A special case of rectangular symmetry and interchangeability is obtained by the circular symmetry assumption, which for f_{IQ} is

$$f_{IQ}(u, v) = h(\sqrt{u^2 + v^2}) \tag{9}$$

for h any non-negative function on $[0, \infty)$. In fact $2\pi r h(r)$ is then the pdf of the envelope samples $(W_{Ii}^2 + W_{Qi}^2)^{1/2}$. Thus circular symmetry is one of the strongest assumptions one can make in extending the univariate zero median and symmetry assumptions, and the zero marginal medians assumption or that of diagonal symmetry is one of the weakest.

NARROWBAND NONPARAMETRIC "SIGN" DETECTORS

Hard-Limiter Narrowband Correlator (HNC) Detectors

The linear narrowband correlator (LNC) detector for a completely known narrowband signal uses the test statistic

$$T_{LNC}(\underline{\tilde{X}}) = \sum_{i=1}^{n} \mathrm{Re}[\tilde{s}_i \tilde{X}_i^*]. \tag{10}$$

Normalization of the $\tilde{X}_i$ to unit envelope values prior to their use in the above correlation gives the hard-limiter narrowband correlator (HNC) test statistic

$$T_{HNC}(\underline{\tilde{X}}) = \sum_{i=1}^{n} \mathrm{Re}[\tilde{s}_i \tilde{X}_i^*/|\tilde{X}_i|]. \tag{11}$$

This is one logical counterpart of the sign detector for a completely known low-pass signal. The HNC detector may be taken to be a nonparametric detector for the class of noise pdf's f_{IQ} which are circularly symmetric. This follows because $\tilde{X}_i/|\tilde{X}_i|$ is then uniformly distributed on the unit circle, and the distribution of $T_{HNC}(\underline{\tilde{X}})$ is fixed and known under H.

The asymptotic relative efficiency (ARE) of the HNC detector relative to the LNC detector for completely known narrowband signal in Gaussian noise can be established to be $ARE_{HNC,LNC} = \pi/4$ [2-4]. For the case where h(r) is the exponential function this ARE becomes 1.5. A general expression of the ARE for a generalized exponent r^k in h(r) is given in [4].

For the incoherent narrowband signal the HNC test statistic may be modified to become

$$T_{HNC}(\underline{\tilde{X}}) = \left| \sum_{i=1}^{n} \tilde{s}_i \tilde{X}_i^*/|\tilde{X}_i| \right|^2 . \tag{12}$$

The resulting detector is also nonparametric for the class of circularly symmetric noise pdf's and its ARE compared to the corresponding LNC detector is the same as in the case of completely known narrowband signals [2-4].

In low-pass known-signal detection the ARE of the sign detector relative to the optimum detector for Gaussian noise is $2/\pi$. The above result of $\pi/4$ indicates better relative performance in the narrowband case, but then the HNC detector may not quite be taken as the counterpart of the low-pass known-signal sign detector.

Narrowband Sign Correlator (NSC) Detectors

As another direct counterpart of the sign detector for a completely known low-pass signal, we can define in the narrowband case the narrowband sign correlator detector based on

$$T_{NSC}(\underline{\tilde{X}}) = \sum_{i=1}^{n} \{s_{Ii} \operatorname{sgn} X_{Ii} + s_{Qi} \operatorname{sgn} X_{Qi}\}$$
$$= \sum_{i=1}^{n} \operatorname{Re}[\tilde{s}_i \tilde{H}_i^*] \qquad (13)$$

where $\tilde{H}_i = (\operatorname{sgn} X_{Ii} + j \operatorname{sgn} X_{Qi})$. In the incoherent case the appropriate test statistic is

$$T_{NSC}(\underline{\tilde{X}}) = |\sum_{i=1}^{n} \tilde{s}_i \tilde{H}_i^*|^2 \qquad (14)$$

In order for these detectors to have constant false-alarm probabilities the class of allowable noise pdf's must be comprised of pdf's f_{IQ} which result in ($\operatorname{sgn} X_{Ii}$, $\operatorname{sgn} X_{Qi}$) having a fixed distribution over its possible values under H. For the class of noise pdf's which give equal-probability quadrants this requirement is fulfilled, and, in fact, $\operatorname{sgn} X_{Ii}$ and $\operatorname{sgn} X_{Qi}$ are independent under H.

It is interesting to note that the narrowband sign detectors defined above can be viewed as versions of the HNC detectors in which the phase angle of $\tilde{X}_i/|\tilde{X}_i|$ is quantized to the values $\pi/4$, $3\pi/4$, $5\pi/4$ and $7\pi/4$ in the first through fourth quadrants, respectively. Such phase quantized detectors have been previously considered [2,3], and their ARE relative to the LNC detectors can be shown to be $2/\pi$ for Gaussian noise. In fact, quantization of the phase angle into more than four levels is possible; under the circular symmetry assumption the resulting detectors will remain nonparametric in their null-hypothesis performance.

NSC Conditional Test Detectors

For the narrowband sign correlator detectors we require assumption (5) in addition to the zero marginal medians assumption in order to establish that they are nonparametric. Here we shall develop nonparametric detectors applicable for narrowband signal detection which operate under the assumption only of zero marginal medians.

In the univariate case for the zero median null hypothesis the sign test is based on partitioning the observation space $\mathbb{R} = (-\infty, \infty)$ into equal probability subsets $(-\infty, 0]$ and $(0, \infty)$. In the present case, under the zero marginal medians null hypothesis, both of the overlapping subsets $\{(u, v) | u \le 0\}$ and $\{(u, v) | v \le 0\}$ have probability 1/2. Thus there are two ways of defining a dichotomous partition of $\mathbb{R}^2$ for the purpose of defining a sign test. Neither one by itself may be useful for the entire range of signal-present situations of interest to us in narrowband signal detection.

One obvious method of partitioning $\mathbb{R}^2$ for a sign test is the partitioning provided by the four quadrants, which we shall now label as C1, D1, C2, D2 for first, second, third and fourth quadrants, respectively. This notation emphasizes that in C1 and C2 the points are concordant (have the same sign for each component) and in D1 and D2 they are discordant. The zero marginal medians condition specifies only that under H $P\{C1 \cup D1\} = P\{C1 \cup D2\} = 1/2$. This allows us to infer that, under H, $P\{C_1\} = P\{C2\}$ and $P\{D1\} = P\{D2\}$, and for the conditional null-hypothesis probabilities of C1, C2, D1 and D2, with $C = C1 \cup C2$ and $D = D1 \cup D2$, that

$$P\{C1|C,H\} = P\{C2|C,H\}$$
$$= 1/2 \qquad (15a)$$

$$P\{D1|D,H\} = P\{D2|D,H\}$$
$$= 1/2 \qquad (15b)$$

These are counterparts of the simple unconditional statement that each half of the real line has probability 1/2 under H in the univariate case.

This observation suggests that it should be possible to use conditional tests to obtain detectors that will be nonparametric in detecting narrowband signals in noise for which f_{IQ} is known only to have zero marginal medians. Consider $T_{NSC}(\underline{\tilde{X}})$ of (13) and (14). We find that in each case the distribution of the components of the $\tilde{H}_i$ is not completely specified under the zero marginal medians null hypothesis. However, conditioned on the value of $M_i = \operatorname{sgn} X_{Ii} \operatorname{sgn} X_{Qi}$ (i.e., given (X_{Ii}, X_{Qi}) is in C or otherwise) we find that $\tilde{H}_i$ is equally likely to be $1 + jM_i$ or $-1 - jM_i$ under this null hypothesis. Thus given the vector $\underline{M} = (M_1, M_2, \ldots, M_n)$ the distribution of $T_{NSC}(\underline{\tilde{X}})$ is fixed and known under the zero marginal medians null hypothesis. We can in fact show that it is sufficient to condition only on the sum

$$M = \sum_{i=1}^{n} M_i$$
$$= N_C - N_D$$
$$= 2N_C - n \qquad (16)$$

or simply N_C, the total number of sign concordances (N_D is the total number of discordances) to make the distributions of $T_{NSC}(\underline{\tilde{X}})$ fixed and known under this H. This follows because given N_C there are $\binom{n}{N_C}$ equally likely possible values of $\underline{M}$ under the null hypothesis.

Thus the implementation of a narrowband sign correlator detector which is nonparametric in its performance under the zero marginal medians null hypothesis requires a stored table of threshold and randomizations which are picked out as functions of M or N_C. The size of the tables grows linearly with n and the individual threshold and randomization values depend only on N_C (or M), the desired false alarm probability and the known vector $\underline{\tilde{s}}$.

Let us consider a useful special case. For the incoherent narrowband signal the test statistic (14) may be written as

$$T_{NSC}(\tilde{\underline{X}}) = \tilde{\underline{s}}\,\tilde{H}^h\,\tilde{H}\,\tilde{\underline{s}}^h \qquad (17)$$

where $\tilde{\underline{s}} = (\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_n)$ and $\tilde{H}$ is similarly defined, with "h" denoting the conjugate-transpose. Now consider the detection of a random phase sinusoidal waveform, so that $\phi(t) = 0$ and $v(t) = 1$ in (1). This means that $s_i = 1 + j0$ so that (17) becomes

$$T_{NSC}(\tilde{\underline{X}}) = \Big(\sum_{i=1}^{n} \operatorname{sgn} X_{Ii}\Big)^2 + \Big(\sum_{i=1}^{n} \operatorname{sgn} X_{Qi}\Big)^2 = \underline{T}_s\,\underline{T}_s^h \qquad (18)$$

where $\underline{T}_s = [T_s(\underline{X}_I), T_s(\underline{X}_Q)]$, the row vector of component sign test statistics for $\underline{X}_I = (X_{I1}, X_{I2}, \ldots, X_{In})$ and for $\underline{X}_Q$. Since the test statistic is to be used in a test conditional on M, one can normalize (18) by the conditional covariance matrix $\underline{V}$ of $\underline{T}_s$. The conditional expected value of $\underline{T}_s$, given M, under H, is $\underline{O}$, and we have

$$\underline{V} = E\{\underline{T}_s^h\,\underline{T}_s | M, H\} = \begin{bmatrix} n & M \\ M & n \end{bmatrix} \qquad (19)$$

This gives as an equivalent to $T_{NSC}(\tilde{\underline{X}})$ of (18) (for conditional testing)

$$T_{BS}(\underline{X}_I, \underline{X}_Q) = \underline{T}_s\,\underline{V}^{-1}\,\underline{T}_s^{\,h} = \frac{(N_{C1} - N_C/2)^2}{N_C/4} + \frac{(N_{D1} - N_D/2)^2}{N_D/4}. \qquad (20)$$

Here N_{C1} is the total number of observations lying in region C1 (first quadrant), etc. This result follows after some algebra, and is easy to interpret as the sum of two-sided sign tests applied separately to observations in C and D. These two separate sign test statistics are conditionally independent, given N_C or N_D (or $M = N_C + N_D$) under H.

The bivariate sign test statistic of (20) was originally developed by Chatterjee [5,6]. In its un-normalized form it has been essentially extended here to the test statistic (17) for the general incoherent narrowband signal detection problem. It turns out that the bivariate sign test statistic of (20) has for $n\to\infty$ an asymptotic conditional distribution, under H and conditioned on N_c, which is the chi-squared (2) distribution. Under an appropriate sequence of alternatives its conditional distribution also converges to a non-central chi-squared distribution which is the same as the unconditional asymptotic distribution. These statements can also be made for the more general bivariate sign correlator and narrowband sign correlator statistic of (17). These results allow the ARE of the nonparametric detector based on the narrowband sign correlator statistic to be obtained relative to the LNC detector; for Gaussian noise the ARE is $2/\pi$.

In order to obtain improved efficiency, it is now possible to use a multi-level test on each of the regions C and D, provided diagonal symmetry can be assumed. The idea is now to subdivide C1 and C2 as $C1 = C1_1 \cup C1_2$ and $C2 = C2_1 \cup C2_2$, for example, with $C1_i \cup C2_i$ forming a region closed under sign changes in both components. The basic idea is similar to that in the univariate case [1]. The problem is that the complexity of the test grows faster, although a simple "dead-zone" type test statistic should require conditioning on only two scalars (numbers of observations in $C1_1 \cup C2_1$ and in $D1_1 \cup D2_1$).

In conclusion we note that it is also not possible to define narrowband signed-rank tests, even under the strong symmetry assumptions considered earlier, without use of conditional tests. Under the simplest assumption of diagonal symmetry it is possible to define conditional bivariate signed-rank tests [6] which may be extended to apply in the general narrowband case. The complexity of such true finite-simple nonparametric signed-rank tests would probably render them impractical for most applications.

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